National Curriculum for
GENERAL MATHEMATICS
Grades IX-X (Humanities Group)
2007

GOVERNMENT OF PAKISTAN
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Contents

Introduction

The Curriculum of General Mathematics for grades IX-X has been designed keeping in view the socio-economic, technical, professional and labour market needs of the country. It continuously focuses on the contents of five Mathematics Standards framed in the National Curriculum of Mathematics 2006. The following themes permeate the curriculum for General Mathematics.

- The curriculum builds the solid conceptual foundation in Mathematics that enables the students to apply their knowledge skillfully and further their learning successfully.

- The curriculum accentuates arithmetic skills that apply to both personal and vocational business opportunities.

- The flourishing fields of financial and business Mathematics have been introduced to provide step-by-step guidance. They serve as a good foundation for further study in management, accounting and finance.

- The curriculum emphasizes on the fundamental concepts of geometry that enable the students to think logically.

- The curriculum stresses graphics that enable the students to visualize and interpret mathematical expressions correctly rather to manipulate them 'blindly'.

National Curriculum of General Mathematics has been developed for humanities group. Upon completion the learner will be able to:

- recognize Mathematics as integral to the development of different cultures and civilizations of the society.

- gain an understanding of financial instruments and terminology used in business finance.

- compare the services offered by a variety of financial institutions.

- familiarize him(her)self with the business partnership, foreign exchange, leasing/financing and life and vehicle insurance.

- apply problem-solving strategies to solve problems encountered by consumers e.g., computation of personal income and utility bills.

- understand real-life situations where calculation of Zakat, Ushr and various taxes is required.

In the curriculum the role of teachers has been rerouted that shifts from 'dispensing information' to planning investigative tasks, managing a cooperative learning environment and supporting students' creativity in developing rational understanding of the concepts of Mathematics.

To ensure that assessment and evaluation are based on curriculum expectations, specific strategies are suggested that lead to the improvement of student learning. An effective learning-outcomes-oriented quality assurance system that is based on constant monitoring and effective feedback loops is recommended.

Print materials, particularly the textbooks, have to play a key role towards providing quality education at all levels. Although there are many stakeholders that contribute towards the overall learning of the child yet the importance of textbook as a reservoir of information/knowledge cannot be ignored. In addition to the textbook, teaching and learning resources include teacher's manual and electronic resources. The guidelines to develop these resources are elaborated.

CURRICULUM FOR GENERAL MATHEMATICS – GRADES IX-X

Contents and Scope	Learning Outcomes /Skills
	All students will be able to

UNIT 1   PERCENTAGE, RATIO AND PROPORTION

1.1 Percentage	i) Know percentage as a fraction with denominator of 100. ii) Convert: • a percentage to a fraction by expressing it as a fraction with denominator 100. • a fraction to a percentage by multiplying it with 100%. • a percentage to a decimal and vice versa. iii) Solve real life problems involving percentage.
1.2 Ratio	i) Know: • a ratio as a relation, which one quantity bears to another quantity of the same kind with regard to their magnitudes. • that, of the two quantities forming a ratio, the first one is called antecedent and the second one consequent. • that a ratio has no units. • the importance of the order in which the ratio is expressed. ii) Find the ratio when a number is increased (decreased) to become another number (e. g., in what ratio must 40 be decreased to become 24?). iii) Solve real life problems involving ratios.
1.3 Proportion	i) Know that an equality of two ratios $\left(\dfrac{a}{b} = \dfrac{c}{d} \right)$ constitutes a proportion, that is, $a:b::c:d$, where a, d

		are known as extremes and *b*, *c* are called the means.
	ii)	Find proportion (direct and inverse).
	iii)	Solve real life problems involving direct and inverse proportion.
1.4 Compound Proportion	i)	Know the concept of compound proportion.
	ii)	Solve real life problems involving compound proportion.

UNIT 2   ZAKAT, USHR AND INHERITANCE

2.1 Zakat	i)	Know 'Nisab', both in 'tola' and gram, on which 'Zakat' is due.
	ii)	Know the rate of Zakat.
	iii)	Calculate amount of Zakat in respect of assets owned by a person.
2.2 Ushr	i)	Know the rate of 'Ushr' levied on land-owner/land-holder in respect of produce of the land.
	ii)	Calculate amount of Ushr in respect of produce of land.
	iii)	Solve real life problems involving Zakat and Ushr.
2.3 Inheritance	i)	Know the ratio of shares among legal inheritors of a property.
	ii)	Calculate amount of share of each legal inheritor of a property.

UNIT 3   BUSINESS MATHEMATICS

3.1 Profit and Loss	i)	Know the
		• cost price (*CP*) as the price, an article is purchased for.
		• selling price (*SP*) as the price, an article is sold for.
	ii)	Identify the following relations regarding profit (when $SP > CP$):
		• Profit $= SP - CP$,

	$SP = \text{Profit} + CP$, $CP = SP - \text{Profit}$, $\text{Profit \%} = \dfrac{\text{Profit}}{CP} \times 100$, $\text{Profit} = \dfrac{CP \times \text{Profit \%}}{100}$, $SP = CP \times \left(\dfrac{100 + \text{Profit \%}}{100}\right)$, $CP = \dfrac{100 \times SP}{100 + \text{Profit \%}}$. iii) Identify the following relations regarding loss (when $SP < CP$): $\text{Loss} = CP - SP$, $SP = CP - \text{Loss}$, $CP = \text{Loss} + SP$, $\text{Loss \%} = \dfrac{\text{Loss}}{CP} \times 100$, $\text{Loss} = \dfrac{CP \times \text{Loss \%}}{100}$, $SP = CP \times \left(\dfrac{100 - \text{Loss \%}}{100}\right)$, $CP = \dfrac{100 \times SP}{100 - \text{Loss \%}}$. iv) Solve real life problems involving profit and loss.
3.2 Discount	i) Recognize marked price (*MP*) or list price of an article. ii) Identify the following relations regarding discount: $\text{Discount} = MP - SP$, $SP = MP - \text{Discount}$, $\text{Discount \%} = \dfrac{\text{Discount}}{MP} \times 100$, $SP = MP \times \left(\dfrac{100 - \text{Discount \%}}{100}\right)$, $MP = \dfrac{100 \times SP}{100 - \text{Discount \%}}$.

	iii)	Solve real life problems involving discount.
3.3 Business Partnership	i)	Know the meaning of business partnership.
	ii)	Distribute the profit among the partners of a partnership concern.
GUIDANCE FOR AUTHOR Include number of partners at most four in the problems of partnership.		

UNIT 4 FINANCIAL MATHEMATICS

4.1 Commercial Banking	i)	Know commercial bank deposit and types of a bank account (PLS savings bank account, current deposit account, PLS term deposit account and foreign currency account).
	ii)	Describe negotiable instruments like cheque, demand draft and pay order.
	iii)	Explain on-line banking, transactions through ATM (Auto Teller Machine), debit card and credit card (Visa and Master).
4.2 Exchange of Currencies		Convert the value of a given amount of the currency of one country in terms of another currency.
4.3 Profit/Markup	i)	Calculate • the profit/markup, • the principal amount, • the profit/markup rate, • the period.
	ii)	Solve problems related to commercial banking and national saving schemes.
4.4 Insurance	i)	Define insurance in its simple terms.
	ii)	Know life insurance and vehicle insurance.
	iii)	Solve simple real life problems regarding purchase of life and motor vehicle insurance.
4.5 Leasing/Financing	i)	Know

	• leasing/financing of motor vehicle, • down payment, • motor vehicle insurance, • processing charges, • repayment in monthly installments. ii) Solve problems related to leasing/financing of motor vehicle under different conditions.

UNIT 5   CONSUMER MATHEMATICS

5.1 Taxes	i) Know the term tax (direct and indirect). ii) Explain the following in simplest possible terms: • sales tax, • excise duty, • property tax, • income tax. iii) Calculate the amount of • sales tax, levied on various commodities, • excise duty, levied on different items, • property tax, imposed on property, • income tax, imposed on an individual with fixed income.
GUIDANCE FOR AUTHOR Introduce the terms financial year, tax year, basic threshold and tax slabs (income tax).	
5.2 Utility Bills	Calculate amount of bill for • electricity, • gas, • telephone, when previous and present meter readings are given.
GUIDANCE FOR AUTHOR (a) Include sales tax and cases of different slabs wherever applicable. For example current slabs for electricity consumed (domestic consumers) are:	

- First 100 units   @ Rs. 2.65,
- Second 200 units   @ Rs. 3.64,
- Next 700 units   @ Rs. 6.15,
- Remaining units   @ Rs. 7.41.

Similarly the slabs for consumption of gas (domestic consumers) may be included.

(b)   For landline telephone bills include the following costs; line rent, local call units/charges, NWD call units/charges, local mobile units/charges, NWD mobile units/charges and excise duty. Exclude the cases for overseas call charges, trunk call charges and PRS charges etc.

(c)   For cellular phones include problems involving call charges, SMS charges and deductions in respect of prepaid and postpaid connections accordingly.

5.3 Personal Income	i)	Calculate personal income (weekly, monthly and annually) of
		• a worker who is paid on daily basis.
		• a worker who is paid for overtime on hourly basis in addition to his daily wages.
		• a salesman who is paid for overtime on hourly basis and commission on different sales in addition to his regular pay.
	ii)	Calculate gross income of a salaried person who is paid on the basis of government pay scales or otherwise.
	iii)	Calculate net income taking into account assorted deductions (income tax etc).

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Include simple allowances wherever necessary; for example house rent allowance and conveyance allowance etc.

UNIT 6   EXPONENTS AND LOGARITHMS

6.1 Radicals and Radicands	i)	Explain the concept of radicals and radicands.
	ii)	Differentiate between radical form and exponential form of an expression.
	iii)	Transform an expression given in radical form to an

		exponential form and vice versa.
6.2 Laws of Exponents/Indices	i)	Recall base, exponent and value.
	ii)	Apply the laws of exponents to simplify expressions with real exponents.
6.3 Scientific Notation		Express a number in standard form of scientific notation and vice versa.
6.4 Logarithm	i)	Define logarithm of a number to the base a as the power to which a must be raised to give the number (i.e., $a^x = y \iff \log_a y = x,\ a > 0,\ y > 0$ and $a \neq 1$).
	ii)	Define a common logarithm, characteristic and mantissa of log of a number.
	iii)	Use tables to find the log of a number.
	iv)	Give concept of antilog and use tables to find the antilog of a number.
6.5 Laws of Logarithm		Prove the following laws of logarithm.
		• $\log_a(mn) = \log_a m + \log_a n$,
		• $\log_a(\frac{m}{n}) = \log_a m - \log_a n$,
		• $\log_a m^n = n\log_a m$.
6.6 Application of Logarithm		Apply laws of logarithm to convert lengthy processes of multiplication, division and exponentiation into easier processes of addition and subtraction, etc.

UNIT 7 ARITHMETIC AND GEOMETRIC SEQUENCES

7.1 Sequence	i)	Define a sequence (progression) and its terms.
	ii)	Know that a sequence can be constructed from a formula or an inductive definition.
7.2 Arithmetic Sequence	i)	Identify arithmetic sequence.
	ii)	Find the n th or the general term of an arithmetic sequence.

	iii)	Solve problems involving arithmetic sequence.
7.3 Arithmetic Mean	i)	Know arithmetic mean between two numbers.
	ii)	Insert n arithmetic means between two numbers.
7.4 Geometric Sequence	i)	Identify a geometric sequence.
	ii)	Find the nth or the general term of a geometric sequence.
	iii)	Solve problems involving geometric sequence.
7.5 Geometric Mean	i)	Know geometric mean between two numbers.
	ii)	Insert n geometric means between two numbers.

UNIT 8   SETS AND FUNCTIONS

8.1 Set		
8.1.1 Operations on Sets	i)	Recall the sets denoted by N, Z, W, E, O, P and Q.
	ii)	Recognize set operations ($\cup, \cap, \setminus, \dots$).
	iii)	Perform the following operations on sets:
		• union,
		• intersection,
		• complement.
8.1.2 Properties of Union and Intersection	iv)	Verify the following fundamental properties of union and intersection of two or three given sets.
		• Commutative property of union and intersection,
		• Associative property of union and intersection.
8.1.3 Venn Diagram	v)	Use Venn diagram to represent
		• union and intersection of sets,
		• complement of a set.
	vi)	Use Venn diagram to verify
		• commutative laws for union and intersection of sets,
		• associative laws for union and intersection of sets.
		• De Morgan's laws.

8.2 Binary Relation	Define binary relation and identify its domain and range.

8.3 Function	i) Define function and identify its domain and range.
	 ii) Demonstrate the following
	      • into function,
	      • one-one function,
	      • into and one-one function (injective function),
	      • onto function (surjective function),
	      • one-one and onto function (bijective function).

GUIDANCE FOR AUTHOR Include sufficient exercises to clarify the concepts of binary relations and functions.	

UNIT 9   LINEAR GRAPHS

9.1 Cartesian Plane and Linear Graphs	i) Identify pair of real numbers as an ordered pair.
	 ii) Recognize an ordered pair through different examples; for instance, an ordered pair $(2,3)$ to represent a seat, located in an examination hall, at the 2nd row and 3rd column.
	 iii) Describe rectangular or Cartesian plane consisting of two number lines intersecting at right angles at the point O.
	 iv) Identify origin (O) and coordinate axes (horizontal and vertical axes or x-axis and y-axis) in the rectangular plane.
	 v) Locate an ordered pair (a, b) as a point in the rectangular plane and recognize:
	      • a as the x-coordinate (or abscissa),
	      • b as the y-coordinate (or ordinate).
	 vi) Draw different geometrical shapes (e.g., line segment, triangle and rectangle, etc.) by joining a set of given points.

	vii)	Construct a table for pairs of values satisfying a linear equation in two variables.
	viii)	Plot the pairs of points to obtain the graph of a given expression.
	ix)	Choose an appropriate scale to draw a graph.
	x)	Draw the graph of
		• an equation of the form $y = c$.
		• an equation of the form $x = a$.
		• an equation of the form $y = mx$.
		• an equation of the form $y = mx + c$.
	xi)	Draw a graph from a given table of (discrete) values.
	xii)	Identify through graph the domain and the range of a function.
GUIDANCE FOR AUTHOR		
Include sufficient exercises to draw linear graphs.		
9.2 Conversion Graphs	i)	Interpret conversion graph as a linear graph relating to two quantities which are in direct proportion.
	ii)	Read a given graph to know one quantity corresponding to another.
	iii)	Read the graph for conversions of the form:
		• miles and kilometres,
		• acres and hectares,
		• degrees Celsius and degrees Fahrenheit,
		• Pakistani currency and another currency, etc.

UNIT 10 BASIC STATISTICS

10.1 Frequency Distribution	i)	Construct grouped frequency table.
	ii)	Construct histograms with equal and unequal class intervals.
	iii)	Construct a frequency polygon.
10.2 Cumulative Frequency	i)	Construct a cumulative frequency table.

Distribution	ii)	Draw a cumulative frequency polygon.
10.3 Measures of Central Tendency	i)	Calculate (for ungrouped and grouped data): • arithmetic mean by definition and using deviations from assumed mean, • median, mode, geometric mean, harmonic mean.
	ii)	Recognize properties of arithmetic mean.
	iii)	Calculate weighted mean and moving averages.
	iv)	Estimate median, quartiles and mode graphically.
10.4 Measures of Dispersion		Measure range, variance and standard deviation.

UNIT 11   ALGEBRAIC FORMULAS AND APPLICATIONS

11.1 Algebraic Expressions	i)	Know that a rational expression behaves like a rational number.
	ii)	Define a rational expression as the quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$ where $q(x)$ is not the zero polynomial.
	iii)	Examine whether a given algebraic expression is a • polynomial or not, • rational expression or not.
	iv)	Define $\frac{p(x)}{q(x)}$ as a rational expression in its lowest terms if $p(x)$ and $q(x)$ are polynomials with integral coefficients and having no common factor.
	v)	Examine whether a given rational algebraic expression is in lowest form or not.
	vi)	Reduce a given rational expression to its lowest terms.
	vii)	Find the sum, difference and product of rational expressions.
	viii)	Divide a rational expression with another and express the result in its lowest terms.
	ix)	Find value of algebraic expression at some particular real number.

11.2 Algebraic Formulas	i)	Know the formulas $$(a+b)^2 + (a-b)^2 = 2(a^2+b^2),$$ $$(a+b)^2 - (a-b)^2 = 4ab.$$ • Find the value of a^2+b^2 and of ab when the values of $a+b$ and $a-b$ are known.
	ii)	Know the formula $$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca.$$ • Find the value of $a^2+b^2+c^2$ when the values of $a+b+c$ and $ab+bc+ca$ are given. • Find the value of $a+b+c$ when the values of $a^2+b^2+c^2$ and $ab+bc+ca$ are given. • Find the value of $ab+bc+ca$ when the values of $a^2+b^2+c^2$ and $a+b+c$ are given.
	iii)	Know the formulas $$(a \pm b)^3 = a^3 \pm 3ab(a \pm b) \pm b^3,$$ $$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2).$$ • Find the value of $a^3 \pm b^3$ when the values of $a \pm b$ and ab are given. • Find the continued product of $$(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2).$$
11.3 Surds and their Application	i)	Recognize the surds and their application.
	ii)	Explain the surds of second order. Use basic operations on surds of second order to rationalize the denominators and evaluate it.
11.4 Rationalization		Explain rationalization (with precise meaning) of real numbers of the types $\frac{1}{a+b\sqrt{x}}$, $\frac{1}{\sqrt{x}+\sqrt{y}}$ and their combinations where x and y are natural numbers and a and b are integers

UNIT 12   FACTORIZATION

12.1 Factorization	Factorize the expressions of the following types.
	Type I: $kx + ky + kz$,
	Type II: $ax + ay + bx + by$,
	Type III: $a^2 \pm 2ab + b^2$,
	Type IV: $a^2 - b^2$,
	Type V: $(a^2 \pm 2ab + b^2) - c^2$,
	Type VI: $a^4 + a^2b^2 + b^4$ or $a^4 + 4b^4$,
	Type VII: $x^2 + px + q$,
	Type VIII: $ax^2 + bx + c$,
	Type IX: $\begin{cases} a^3 + 3a^2b + 3ab^2 + b^3, \\ a^3 - 3a^2b + 3ab^2 - b^3, \end{cases}$
	Type X: $a^3 \pm b^3$.
12.2 Remainder Theorem and Factor Theorem	i) State and apply remainder theorem. ii) Find remainder (without dividing) when a polynomial is divided by a linear polynomial. iii) Define zeros of a polynomial. iv) State factor theorem and explain through examples.
12.3 Factorization of Cubic Polynomial	Use factor theorem to factorize a cubic polynomial.

UNIT 13   ALGEBRAIC MANIPULATION

13.1 Highest Common Factor and Least Common Multiple	i) Find highest common factor (HCF) and least common multiple (LCM) of algebraic expressions. ii) Use factor or division method to determine HCF and LCM. iii) Know the relationship between HCF and LCM.
13.2 Basic Operations on Algebraic Fractions	Use HCF and LCM to reduce fractional expressions involving $+, -, \times, \div$.

13.3 Square Root of Algebraic Expression	Find square root of an algebraic expression by factorization and division.

UNIT 14   LINEAR EQUATIONS AND INEQUALITIES

14.1 Linear Equations	i) Recall linear equation in one variable. ii) Solve linear equation with rational coefficients. iii) Reduce equations, involving radicals, to simple linear form and find their solutions.
14.2 Equation involving Absolute Value	i) Define absolute value. ii) Solve the equation, involving absolute value, in one variable.
14.3 Linear Inequalities	i) Define inequalities $(>, <)$ and $(\geq, \leq)$. ii) Recognize properties of inequalities (i.e. trichotomy, transitive, additive and multiplicative).
14.4 Solving Linear Inequalities	Solve linear inequalities with rational coefficients.

UNIT 15   QUADRATIC EQUATIONS

15.1 Quadratic Equation	Define quadratic equation.
15.2 Solution of Quadratic Equations	Solve a quadratic equation in one variable by • factorization, • completing square.
15.3 Quadratic Formula	i) Use method of completing square to derive quadratic formula. ii) Use quadratic formula to solve quadratic equations. iii) Solve simple real life problems.

UNIT 16   MATRICES AND DETERMINANTS

16.1 Introduction to Matrices	Define • a matrix with real entries and relate its rectangular layout (formation) with real life, • rows and columns of a matrix, • the order of a matrix, • equality of two matrices.
16.2 Types of Matrices	Define and identify row matrix, column matrix, rectangular matrix, square matrix, zero/null matrix, identity matrix, scalar matrix, diagonal matrix, transpose of a matrix, symmetric and skew-symmetric matrices.
16.3 Addition and Subtraction of Matrices	i) Know whether the given matrices are conformable for addition/subtraction. ii) Add and subtract matrices. iii) Multiply a matrix by a real number. iv) Verify commutative and associative laws under addition. v) Define additive identity of a matrix. vi) Find additive inverse of a matrix.
16.4 Multiplication of Matrices	i) Know whether the given matrices are conformable for multiplication. ii) Multiply two (or three) matrices. iii) Verify associative law under multiplication. iv) Verify distributive laws. v) Show with an example that commutative law under multiplication does not hold in general (i.e., $AB \neq BA$). vi) Define multiplicative identity of a matrix. vii) Verify the result $\left(AB\right)^{t} = B^{t}A^{t}$.
16.5 Multiplicative Inverse of a Matrix	i) Define the determinant of a square matrix. ii) Evaluate determinant of a matrix.

	iii)	Define singular and non-singular matrices.
	iv)	Define adjoint of a matrix.
	v)	Find multiplicative inverse of a non-singular matrix A and verify that $AA^{-1} = I = A^{-1}A$, where I is the identity matrix.
	vi)	Use adjoint method to calculate inverse of a non-singular matrix.
	vii)	Verify the result $(AB)^{-1} = B^{-1}A^{-1}$.
16.6 Solution of Simultaneous Linear Equations	Solve a system of two linear equations and related real life problems in two unknowns using • Matrix inversion method, • Cramer's rule.	

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- Order of a matrix may be written as *m*–by–*n* instead of $m \times n$.
- A matrix of order at most 3–by–3 may be used while defining (considering) different types of matrices, their addition and scalar multiplication etc.
- Order of the matrix may be restricted to 2–by–2 when matrix multiplication, determinant, adjoint, inverse and system of equations are considered.

UNIT 17   FUNDAMENTALS OF GEOMETRY

17.1 Properties of Angles	i)	Define adjacent, complementary and supplementary angles.
	ii)	Define vertically-opposite angles.
	iii)	Calculate unknown angles involving adjacent angles, complementary angles, supplementary angles and vertically opposite angles.
	iv)	Calculate unknown angle of a triangle.
17.2 Parallel Lines	i)	Define parallel lines.
	ii)	Demonstrate through figures the following properties of parallel lines. • Two lines which are parallel to the same given line

		are parallel to each other.
		• If three parallel lines are intersected by two transversals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are also equal.
		• A line through the midpoint of a side of a triangle parallel to another side bisects the third side (an application of above property).
	iii)	Draw a transversal to intersect two parallel lines and demonstrate corresponding angles, alternate-interior angles, vertically-opposite angles and interior angles on the same side of transversal.
	iv)	Describe the following relations between the pairs of angles when a transversal intersects two parallel lines:
		• Pairs of corresponding angles are equal,
		• Pairs of alternate interior angles are equal,
		• Pair of interior angles on the same side of transversal is supplementary,
		and demonstrate them through figures.
17.3 Congruent and Similar Figures	i)	Identify congruent and similar figures.
	ii)	Recognize the symbol of congruency.
	iii)	Apply the properties for two figures to be congruent or similar.
17.4 Congruent Triangles		Apply following properties for congruency between two triangles.
		• $SSS \cong SSS$,
		• $SAS \cong SAS$,
		• $ASA \cong ASA$,
		• $RHS \cong RHS$.
17.5 Quadrilaterals	i)	Demonstrate the following properties of a square.
		• The four sides of a square are equal.
		• The four angles of a square are right angles.

	• Diagonals of a square bisect each other and are equal.
	
	ii) Demonstrate the following properties of a rectangle.
	
	    • Opposite sides of a rectangle are equal.
	
	    • The four angles of a rectangle are right angles.
	
	    • Diagonals of a rectangle bisect each other.
	
	iii) Demonstrate the following properties of a parallelogram.
	
	    • Opposite sides of a parallelogram are equal.
	
	    • Opposite angles of a parallelogram are equal.
	
	    • Diagonals of a parallelogram bisect each other.

17.6 Circle	i) Describe a circle and its centre, radius, diameter, chord, arc, major and minor arcs, semicircle and segment of the circle.
	
	ii) Describe the terms; sector and secant of a circle, concyclic points, tangent to a circle and concentric circles.
	
	iii) Demonstrate the following properties:
	
	    • the angle in a semicircle is a right angle,
	
	    • the angles in the same segment of a circle are equal,
	
	    • the central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.
	
	iv) Apply the above properties in different geometrical figures.

UNIT 18   PRACTICAL GEOMETRY

18.1 Construction of Triangle	i) Construct a triangle having given:
	
	    • two sides and the included angle,
	
	    • one side and two of the angles,
	
	    • two of its sides and the angle opposite to one of them (with all the three possibilities).
	
	ii) Draw:
	
	    • angle bisectors,

	• altitudes, • perpendicular bisectors, • medians, of a given triangle and verify their concurrency.

18.2 Construction of Quadrilateral	i) Construct a rectangle when:
	       • two sides are given.
	       • diagonal and one side are given.
	ii) Construct a square when its diagonal is given.
	iii) Construct a parallelogram when two adjacent sides and the angle included between them is given.

18.3 Tangent to the Circle	i) Locate the centre of a given circle.
	ii) Draw a circle passing through three given non-collinear points.
	iii) Draw a tangent to a given circle from a point P when P lies
	       • on the circumference,
	       • outside the circle.
	iv) Draw
	       • direct common tangent or external tangent,
	       • transverse common tangent or internal tangent
	       to two equal circles.
	v) Draw
	       • direct common tangent or external tangent,
	       • transverse common tangent or internal tangent
	       to two unequal circles.
	vi) Draw a tangent to
	       • two unequal touching circles,
	       • two unequal intersecting circles.

UNIT 19 AREAS AND VOLUMES

19.1 Pythagoras Theorem	i)	State Pythagoras theorem.
	ii)	Solve right angled triangle using Pythagoras theorem.
19.2 Areas	i)	Find the area of
		• a triangle when three sides are given (apply Hero's formula),
		• a triangle whose base and altitude are given,
		• an equilateral triangle when its side is given,
		• a rectangle when its two sides are given,
		• a parallelogram when base and altitude are given,
		• a square when its side is given,
		• four walls of a room when its length, width and height are given.
	ii)	Find the cost of turfing a square/rectangular field.
	iii)	Find the number of tiles, of given dimensions, required to pave the footpath of given width carried around the outside of a rectangular plot.
	iv)	Find the area of a circle and a semi circle when radius is given.
	v)	Find the area enclosed by two concentric circles whose radii are given.
	vi)	Solve real life problems related with areas of triangle, rectangle, square, parallelogram and circle.
19.3 Volumes	i)	Find the volume of
		• a cube when its edge is given.
		• a cuboid when its length, breadth and height are given.
		• a right circular cylinder whose base radius and height are given.
		• a right circular cone whose radius and height are known.
		• a sphere and a hemisphere when radius is given.

	ii) Solve real life problems related to volume of cube, cuboid, cylinder, cone and sphere.

UNIT 20   INTRODUCTION TO COORDINATE GEOMETRY

20.1 Distance Formula	i) Define coordinate geometry.
	ii) Derive distance formula to find distance between two points given in Cartesian plane.
	iii) Use distance formula to find distance between two given points.
20.2 Collinear Points	i) Define collinear points.
	ii) Distinguish between collinear and non-collinear points.
	iii) Use distance formula to show that given three (or more) points are collinear.
	iv) Use distance formula to show that the given three non-collinear points form:
	• an equilateral triangle,
	• an isosceles triangle,
	• a right angled triangle,
	• a scalene triangle.

Teaching Strategies

◼ Introduction

In the classrooms teachers transmit textbook facts to students, who in turn are expected to memorize and regurgitate them in examinations. The teachers are so ingrained that they find this method of lecture and recitation as a good way of teaching a large number of students in their classrooms. In Mathematics students memorize rules without understanding their rationale. There is no doubt that the timely reward to this way is more immediate and more apparent but this instrumental learning does not help subsequently. The memorized rules may work for a limited range of similar problems but students do not feel comfortable when they face different or challenging tasks. Consequently the students are totally dependent upon teachers. They cannot progress in thinking and their self-esteem therefore is low.

To capture all aspects of expertise, competence, knowledge and facility which are necessary for anyone to learn Mathematics, Kilpatrick et al[†] (2001) present the notion of mathematical proficiency that is composed of following five interwoven but interdependent strands:

- **Conceptual understanding** – comprehension of mathematical concepts, operations and relations.
- **Procedural fluency** – skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
- **Strategic competence** – ability to formulate, represent and solve mathematical problems.
- **Adaptive reasoning** – capacity for logical thought, reflection, explanation and justification.
- **Productive disposition** – habitual inclination to see Mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy.

◼ Part I Teaching Mathematics – Role of Teacher

Research indicates that teachers who have a good background in Mathematics also add richness to their lessons, involve students extensively in mathematical dialogue and capitalize on students' questions/discussions to weave/extend mathematical relationships. They do not list only the definitions and step-by-step procedures for students to memorize without understanding their meaning and function.

Teachers need to assume a new role if students are to construct their own mathematical understanding. Rather than just pouring mathematical knowledge into students' heads,

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[†] Kilpatrick, J., Swafford, J., and Findell, B. (Eds) (2001) *Adding It Up: Helping Children Learn Mathematics*, Mathematics Learning Study Committee, Centre for Education, Washington, DC: National Academies Press.

teachers must create a stimulating environment that encourages mathematical learning through increasing interactivity.

Teaching in a Mathematics classroom requires listening to the students, understanding their level of thinking, setting a task and analysing outcomes of the task in order to understand how students construct meanings – this is contrary to a traditional way of teaching. The teachers' role shifts from dispensing information to planning investigative tasks, managing a cooperative learning environment and supporting students' creativity in developing rational understanding of the concepts. This improved teaching practice should include the following aspects of a teacher's role. The teacher should be

- a planner of practical tasks for the students to consolidate and organise their informal knowledge.
- an organizer of the establishment of mathematical tasks in the classroom so that the students can work in a social setting and develop rational understanding.
- an encourager who asks questions, supports and develops students' mathematical thinking and communication.
- a negotiator helping students to discuss various meanings/solutions of a concept/question and to achieve a common agreement.
- a mediator supporting the establishment of an environment where students express opinions and experiences in the classroom equally.

Thus a teacher's primary responsibilities are to assist learners' cognitive reconstruction and conceptual re-organisation through providing them the opportunities for interaction in mathematical tasks that encourage discussion and negotiation of ideas to help them develop conceptual understanding.

Part II Effective Teaching Strategies

Students learn things in many different ways. They do not always learn best by sitting and listening to the teacher. Students particularly of the primary level can learn by presentation and explanation by the teacher, consolidation and practice, games, practical work, problems and puzzles, and investigating Mathematics.

Investigating Mathematics

Teachers may set students a challenge, matched to their ability, which leads them to discover and practice some new Mathematics for themselves. The key point about investigations is that students are encouraged to make their own decisions about:

- where to start,
- how to deal with challenge,
- what Mathematics they need to use,
- how they can communicate this Mathematics,
- how to describe what they have discovered.

Problem Solving

A problem is a statement or proposition requiring an algebraic, geometric, or other mathematical solution. A widespread opinion that problem solving should be the central focus of the curriculum for Mathematics strengthens the fact that 'learning to solve problems is the principal reason for studying Mathematics'.

A problem exists when there is a situation a learner wants to resolve but no solution is readily apparent. For example Shahzaib counted 19 cycle-wheels, run by 7 cycle-riders,

going past his house. How many tricycles were there? Working on this problem offers a good practice in addition, multiplication and division skills. But the important goal of this problem is to help students think systematically about possibilities and record thinking.

Research suggests that a problem solver needs to become better acquainted with a problem and works for a clearer understanding of it before progressing towards a solution. The path from understanding the problem to devising a plan may sometimes be long but experience and practice are the best teachers to contrive. The plan gives a general outline of direction to solve the problem. Having arrived at a result, it is verified by referring back to the original problem.

Part III Time Distribution

Teaching schedules are among the integral parts of Mathematics classrooms. They help school management to run and monitor the teaching of a particular subject. The following tables, indicating unit-wise time distribution, will be supportive to the teachers and education planners.

UNITWISE TIME DISTRIBUTION – GRADE IX

Unit	Title	Periods (40 minutes each)
1.	Percentage, Ratio and Proportion	21
2.	Zakat, Ushr and Inheritance	16
3.	Business Mathematics	21
4.	Financial Mathematics	21
5.	Consumer Mathematics	26
6.	Exponents and Logarithms	20
7.	Arithmetic and Geometric Sequences	22
8.	Sets and Functions	16
9.	Linear Graphs	26
10.	Basic Statistics	21
	TOTAL	210 (6 periods per week)

UNITWISE TIME DISTRIBUTION – GRADE X

Unit	Title	Periods (40 minutes each)
11.	Algebraic Formulas and Applications	23
12.	Factorization	27
13.	Algebraic Manipulation	15
14.	Linear Equations and Inequalities	17
15.	Quadratic Equations	13
16.	Matrices and Determinants	23
17.	Fundamentals of Geometry	33
18.	Practical Geometry	23
19.	Areas and Volumes	18
20.	Introduction to Coordinate Geometry	18
	TOTAL	210 (6 periods per week)

Assessment and Evaluation

◾ Introduction

Assessment is the process of gathering information using a variety of tools and techniques that reflect how well a student is achieving the curriculum expectations in a subject. As part of assessment teachers provide students with descriptive feedback that guides their efforts towards improvement. The quality of assessment largely determines the quality of evaluation. Evaluation refers to the process of judgments and decisions based on the interpretation of evidence gathered through assessment.

Rowntree[‡] (1990) defined assessment as having two purposes: firstly to support and provide feedback to learners and improve their ongoing learning, and secondly to report on what they had already achieved. In essence the first is formative assessment and the second is summative assessment. Morgan and O'Reilly[§] (1999) believe that assessment is the engine that drives and shapes learning rather than an end of course event that grades and reports on performance.

Assessment and evaluation should be based on curriculum expectations and the achievement levels outlined in the national curriculum. To ensure that assessment and evaluation lead to the improvement of student learning, teachers must use specific assessment and evaluation strategies

- that address both what students learn and how well they learn.
- that are administered over a period of time and designed to provide opportunities for students to demonstrate full range of their learning.
- that ensure that each student is given clear directions for improvement.
- that promote students' ability to assess their own learning.
- that are communicated clearly to students and parents in advance.

Part I   Assessment in Mathematics

It should be kept in mind that in Mathematics a single type of assessment can frustrate students, diminish their self-confidence and make them feel anxious about the subject. In reality the understanding of mathematical concepts encompasses a broad range of abilities. Examples of various templates, to assess different abilities, are mentioned below.

Assessment must include by focusing on a student's ability to:
- communicate mathematically.

[‡] Rowntree, D. (1990) *Teaching through Self-Instruction (Second Ed)*, London: Kogan Page.
[§] Morgan, C. and O'Reilly, M. (1999) *Assessing Open and Distance Learners*, London: Kogan Page.

- reason and analyze, and to think and act in positive ways.
- comprehend the key concepts.
- evaluate the effectiveness of using different strategies to address the same problem.
- use a variety of strategies to problem solving and to make mathematical connections.
- discriminate between relevant and irrelevant attributes of a concept in selecting examples.
- integrate and to make sense of mathematical concept and procedure.
- examine real life situations by reasoning mathematically.

Learning of Mathematics, being a cumulative process, occurs as experiences contribute to understanding. Suggested below are the assessment strategies to obtain valid and reliable picture of students' understanding and achievement.

i) **Classroom-based assessments** that include anecdotal records, checklists, rating scales, portfolios peer- and self-assessment.
ii) **Teacher-designed test formats** that include oral examination, assignments, short answers, matching, multiple-choice, fill-in and true-false items.

Part II The Traditional Examinations

For assessment and evaluation of grades below secondary level, the institutions adopt their own criteria. The means by which each institution achieves quality should differ according to the circumstances in which it operates, but each must give priority to meeting students' expectations in terms of learning outcomes they can legitimately expect to achieve. In essence an effective learning-outcomes-oriented quality assurance system must be based on constant monitoring and effective feedback loops.

Bearing in mind the requirement of simplicity in considering assessment strategies, the examinations in traditional paper-based mode with place and time-specific activities, are easy to organize for institutions (Boards of Intermediate and Secondary Education). When a formal examination, for Secondary School Certificate (SSC) or Higher Secondary School Certificate (HSSC), is used for assessment there are examination centres, infrastructure to supply and secure examination papers before examination and arrangements to check the identities of the candidates, invigilate the examination and collect the scripts for marking. Marks are then gathered and results are promulgated in a timely manner.

Instructions for SSC Examination

The examining institutions or bodies including all Boards of Intermediate and Secondary Education for the conduct of SSC examination in the subject of Mathematics should follow instructions as given below.

(1) The question papers should be balanced in all respect. Following table, showing weightage to difficulty level of questions, is suggested to be a practicable criterion for a balanced question paper of Mathematics.

Difficulty Level of Questions	Weightage
Easy	15 %
Average	70 %
Difficult	15 %

(2)   According to the new Scheme of Studies 150 marks have been allocated to the subject of General Mathematics for SSC Examination. There will be two papers (Paper-A and Paper-B) of General Mathematics each carrying 75 marks.

- General Mathematics **Paper-A** will cover units 1-10.
- General Mathematics **Paper-B** will cover units 11-20.

The examiners will set the question papers keeping in view the following table showing unit-wise weightage of every topic.

UNITWISE WEIGHTAGE – PAPER A

Unit	Title	Weightage
1.	Percentage, Ratio and Proportion	8%
2.	Zakat, Ushr and Inheritance	10%
3.	Business Mathematics	10%
4.	Financial Mathematics	10%
5.	Consumer Mathematics	12%
6.	Exponents and Logarithms	8%
7.	Arithmetic and Geometric Sequences	10%
8.	Sets and Functions	10%
9.	Linear Graphs	12%
10.	Basic Statistics	10%
	TOTAL	100%

UNITWISE WEIGHTAGE – PAPER B

Unit	Title	Weightage
11.	Algebraic Formulas and Applications	12%
12.	Factorization	10%
13.	Algebraic Manipulation	6%
14.	Linear Equations and Inequalities	8%
15.	Quadratic Equations	6%
16.	Matrices and Determinants	12%
17.	Fundamentals of Geometry	14%
18.	Practical Geometry	10%
19.	Areas and Volumes	12%
20.	Introduction to Coordinate Geometry	10%
	TOTAL	100%

Teaching and Learning Resources

◼ Introduction

The government prescribed textbook is the only teaching and learning tool used in most of the Pakistani schools. Though many other resources are also available, accessible and affordable teachers rarely use them to support the learning. In addition to the textbook, the teaching and learning resources include teacher's manual and electronic resources.

Part I The Textbook

There are many important entities involved to revamp the entire education system. The school has to play its own role, parents have to contribute their share and teachers have to assume a significant place in fostering education. Print materials, particularly the textbooks, have to play a key role towards providing quality education at all levels. Although there are many stakeholders that contribute towards the overall learning of child yet the importance of textbook as a reservoir of information/ knowledge cannot be ignored.

Textbook writers have a vital role to play in penetrating the young minds through their writing. A textbook
- whose content as well as presentation is thoughtfully planned,
- which is written by qualified and competent subject expert(s), and
- which is attractive and engaging,

must stimulate the interest of teacher and the taught.

Guidelines for Textbook Authors

Textbooks aimed at lower level tend to include more learning features than those at higher level. However in textbook writing generally the following aspects may be taken into consideration.

- The textbook should be in line with the objectives of National Curriculum.
- The author should bring himself to the mental level of students he is writing for.
- The span of the textbook should be fairly reasonable.
- The material should not be cramped. To make it more digestible, it may be chunked into smaller parts with headings.
- The textbook is expected to provide accurate and up-to-date information.
- The text material should be arranged in a logical manner; simple to complex, familiar to unfamiliar and concrete to abstract.
- The text material must be free from ambiguities and errors (both mathematical and typographical).
- The content provided in the textbook should not develop wrong concepts.

- The text should be clear and concise. It should not give any other meaning than the one intended.
- Special attention should be paid to geometrical portions. Every table, line drawing and graph should be labeled appropriately.
- Footnotes and side notes may be inserted wherever necessary.

Textbook Style and Structure

To make a textbook an effective teaching and learning tool its style and structure is given due importance. The material needs to be structured in a coherent and logical way and that writing style should be reader friendly.

Unit Opening	
Unit Outline	Include list of headings.
Student Learning Outcomes (SLOs)	One SLO for each heading may be included. If they are numerous then a reasonable number is acceptable.
Real Life Relevance	Illustrate the real life relevance of the unit, if possible.
Short Introduction	Explain what this unit covers and why.

Unit Body	
Key Terms	Use italics for emphasis and bold for key terms. Define key terms when first introduced and collate them with their definitions for the glossary.
Running Glossary	Key terms and definitions may be pulled out from the main body of text so that students spot them easily in the unit body (e.g. in the margins).
Feature Boxes	Regular feature boxes may include various contents such as a mathematical formula, a working rule or a statement of theorem.
Illustrative Examples	Include illustrative examples to develop conceptual understanding of the topic.
Problem Sets	Special attention should be paid on preparation of Problem Sets. Correlate Mathematics with real life situations and include sufficient exercises on real life problems almost in every problem set, if appropriate. The questions on the application of Mathematics in other fields of study are also very useful.
Learning Review Points	Include bulleted questions for students to check their understanding at regular intervals. Possible labels include 'self-test point' or 'checkpoint'.
Tips or Hints	Separated from the main body of text, they allow the author to speak directly to the student, offering useful advice or flagging important points.
Visuals	Tables, graphs, line drawings and lists may be used to break up the text.

Unit Ending	
Problem Set (Review)	Include multiple-choice questions, interpretive exercises and fill-in items. Students may also be asked to label diagrams or write a one word answer to short question.
Summary	Include a review of the main concepts. This can relate to the SLOs by covering each in turn (bullet points work well). The summary should not include any new information.

End of Textbook	
Glossary	Include only the key terms in the glossary.
Answers to Problems	Include answers to the problem sets unit wise.
Appendices	Include extra information the student needs such as list of mathematical formulas, log tables and relevant websites.
Bibliography	Include bibliography and list of books for suggested reading where appropriate.
Index	Include index for the key terms used in the book.

Additional Instructions for Authors

A few additional but important instructions for authors are given below.

- There will be two books published for grades IX-X.
- The book titled as **General Mathematics-I** will cover the units 1-10 while the units 11-20 will be swathed in the book titled as **General Mathematics-II**.
- The books General Mathematics-I and General Mathematics-II will be taught at grade level IX and grade level X respectively.

Part II The Teacher's Manual

Ideally a teacher's manual should come with the textbook. The manual is aimed at informing teachers of how the textbook is written and how best to use it to facilitate student learning. It can be seen as a means of helping teachers develop professionally. It provides detailed explanation of key concepts and way to teach a particular topic. Its basic features are as below.

A teacher's manual should
- be easy to understand and use.
- help teachers teach text and extend activities.
- give sequenced instructions for each activity.
- include teaching learning resources.
- establish a question bank (having questions different from text) and suggest interactive quizzes corresponding to each unit.
- involve various up-to-date and relevant teaching strategies and rationale for suggested teaching.

- explain how to implement each teaching strategy.
- identify constraints and strengths of each strategy or activity.
- identify resources needed for teaching strategies and extension of activities.
- expand and develop teachers repertoire of knowledge and skills.
- identify assessment strategies.

Part III   The Web-based Resources

The use of world wide web (www) is growing very fast to access an immense volume of rapidly evolving information. It is acting as a driving force since its ease of use makes the internet trivially accessible to the students even with a little knowledge of computer. Through web-based links as mentioned below the teachers, parents and students will be able to
- access various sites of Mathematics around the world,
- view three-dimensional figures, graphics, lesson plans, activities and various books of interest.

Title of Website	Universal Resource Locator (URL)
A+Math	http://www.aplusmath.com/
AAA Math	http://www.aaamath.com/
Academic Info-Mathematics	http://www.academicinfo.net/math.html
Algebra Buster	http://www.algebra-online.com/
Algebra Helper	http://www.algebrahelp.com/index.jsp
Class Zone	http://www.classzone.com/math_middle.cfm
Click on Bricks	http://kathyschrock.net/clickonbricks/index2.htm
Cool Math	http://www.coolmath.com/
Discovery School (Mathematics)	http://school.discovery.com/lessonplans/math.html
Frank Potter's Science Gems-Mathematics	http://www.sciencegems.com/math.html
Funbrain	http://www.funbrain.com/numbers.html
Geometry	http://www.mathleague.com/help/geometry/geometry.htm
Internet Mathematics Library	http://www.mathforum.org/library
Math Archives	http://www.archives.math.utk.edu/
Math Glossary	http://www.harcourtschool.com/glossary/math_advantage
Math Goodies	http://www.mathgoodies.com
Math World	http://www.mathworld.wolfram.com
Math2	http://www.math2.org/
Mathematical Interactivities	http://mathematics.hellam.net/
MathStories	http://www.mathstroies.com
Mega Mathematics	http://www.c3.lanl.gov/mega-math/
Purplemath	http://www.purplemath.com/internet.htm
S.O.S. Mathematics	http://www.sosmath.com
Superkids Educational Software Review	http://www.superkids.com/aweb/tools/math/index.shtml
Teaching madeEasier	http://www.teachingmadeasier.com/math.html
Webmath	http://www.webmath.com/

Curriculum Development Team

■ Curriculum Writers

Professor Dr Muhammad Tahir
Chairman(Sciences)/Focal Person
National Curriculum Council
Ministry of Education
ISLAMABAD

Dr Malik Muhammad Yousaf
Assistant Professor
Department of Mathematics
Quaid-i-Azam University
ISLAMABAD

■ National Advisory Committee

Professor Dr Muhammad Tahir
Chairman(Sciences)/Focal Person
National Curriculum Council
Ministry of Education, ISLAMABAD

Professor Dr Muneer A Rashid
Centre for Advanced Mathematics & Physics
College of E & ME Campus (NUST)
Peshawar Road, RAWALPINDI

Professor Dr Muhammad Aslam Noor
Department of Mathematics
COMSATS Institute of Information
Technology, ISLAMABAD

Professor Dr Muhammad Aslam Butt
Department of Mathematics
Lahore University of Management
Sciences (LUMS), LAHORE CANTT

Professor Dr Lala Rukh Kamal
Head of Mathematics Department
University of Baluchistan
QUETTA

Professor Dr Gulzar Ali Khan
Chairman Mathematics Department
University of Peshawar
PESHAWAR

Professor Dr Syed Arif Kamal
Department of Mathematics
University of Karachi
KARACHI

Professor Dr Muhammad Shafiq Baig
Chairman Mathematics Department
University of Azad Jammu & Kashmir
MUZAFFARABAD (AJ&K)

Dr Malik Muhammad Yousaf
Assistant Professor
Department of Mathematics
Quaid-i-Azam University, ISLAMABAD